CHOICES OF COMPUTATIONAL STRATEGIES

Paul Swan
Edith Cowan University
<p.swan@cowan.edu.au>

Jack Bana
Edith Cowan University
<j.bana@cowan.edu.au>

This study was designed to explore the computational choices made by 25 students in Years 5-7. The data was collected in individual interviews and the results showed that many students have a very limited repertoire of methods at their disposal, Students often tended to lack confidence in their approach and often it was only then that they reached for a calculator. A number of students used two or more computational methods for some items.

BACKGROUND

There is much debate around the following issues.

- What is the place of standard written algorithms in the classroom?
- What computational choices are sensible in today's calculator age?
- What does it mean to make efficient and effective computational choices?

A National Statement on Mathematics for Australian Schools (1991) indicated that a goal for students in both Band A and Band B was to be able to "choose computational methods (mental, paper and pencil, calculator) and check the reasonableness of results" (pp. 115, 121). This raises the question as to how adept students are at choosing an appropriate computational method with which to solve a problem.

There does seem to be general agreement that students should be taught to make sensible choices among mental, written and calculator methods of computation. This study was designed to investigate the choices made by students in Years 5, 6 and 7 when given a variety of computation tasks. In particular the researchers were interested to discover whether students actually had a choice of computation method and, if so, how they exercised that choice.

Previous studies (Reys, Reys & Hope, 1993; Price, 1997; Carraher, Carraher & Schlieman, 1985) have also focussed on the choices made by students, but not on whether the students had any alternative strategies at their disposal.

Major policy documents and curricula in recent times have promoted the goal that students should learn to make sensible computational choices (AEC, 1990; NCTM, 1989). The issue of computational choice is a complex one. Coburn (1989) has suggested that the notion of computation needs to be broadened to better reflect the variety of choices available to students.

Price (1997) studied teacher presence as a variable in computational choice and found that students favoured the use of paper and pencil over other methods of calculation. It is very interesting to note that with the teacher present, students chose written methods 56 percent of the time, calculators 26 percent, and mental methods 19 percent of the time. Yet a 10 percent swing from written to calculator methods occurred when the teacher left the room. An earlier study (Reys, Reys & Hope, 1993) found similar trends in terms of written algorithms dominating computational choice, but found a higher incidence of mental computation and less use of calculators. They also noted that computational choices varied according to the nature of the numbers used in each computation item.

The preference for choosing written algorithms is probably related to the amount of time spent in school classrooms teaching standard written algorithms (Porter, 1989). Price's (1997) findings that students choose to use written algorithms more often when teachers are nearby than when they are not present, again supports the influence of the teacher on the students' choices here. Carraher, Carraher and Schlieman (1985) found that the setting affected computational choice. In an out-of-school setting the students used self-taught methods whereas in the school setting they chose to use school-taught methods.

Ruthven (1995) has argued that attitudes and beliefs play a role in student computational choice, in particularly the propensity to use a calculator. He notes that "... their degree of scepticism about the legitimacy and beneficence of the calculator and of lack of confidence in the calculator mode of calculation affect students' computational choices" (p. 229). To suggest that there is a clearcut choice between the various computational alternatives of mental, paper and pencil and calculator is too simplistic. Ruthven (1998) who studied the use of mental, written and calculator strategies by upper primary students noted that "... a refinement of the common-sense trichotomy between mental, written and calculator methods was necessary ..." (pp. 29-30). Several models have been proposed to describe the computation process (NCTM, 1989; Coburn, 1989; Trafton, 1994). However, these are linear in structure and imply that students tend to focus on only one method of computation at any time, such as calculator use for example. Swan and Bana (1998) developed a model to represent situations where students use a combination of two or three strategies for a particular computation.

THE STUDY

This study investigated computational methods and computational choice among 25 Year 5, 6 and 7 students spread across two classes in a primary school in Western Australia. Each student was asked to undertake a series of eighteen computation exercises which were presented both in oral and written form (in horizontal format). This was carried out in an individual interview situation, and the student was free to work mentally, or use the pencil and paper or calculator provided. For each item the initial choice of computation method was noted, along with whether or not the student obtained the correct solution. The student was then asked if he/she could answer the question using another method. If the student indicated he/she could solve the problem in another way then the interviewer requested he/she demonstrate this. Records were kept of successful and unsuccessful attempts, and of computational preference. Previous studies have focussed on computational choice but not on whether students had alternative choices at their disposal nor whether they used a combination of methods to solve a particular problem. Students were questioned to determine whether their initial choice was their only choice, and they were also asked to explain their methods of solution.

The Instrument

The instrument used to determine students' computational preferences was developed as a result of a pilot study (Swan and Bana, 1998) and also from previous studies relating to computational preference (Reys, Reys & Hope, 1993; Price, 1995). The instrument is shown in Table 1 and consisted of 18 computation items. All but two of these were presented out of context in order to focus on the computation by eliminating associated extraneous variables.

Table 1
The Eighteen-Item Test Instrument

Number	Item	Number	Item
1	28 + 37	10	14 x 9 ÷ 6
2	74 – 36	11	$^1/_2 + ^3/_4$
3	369 ÷ 3	12	$10 - 4^3/_4$
4	36 x 25*	13	$^2/_3$ of 45
5	70 x 600*	14	$1.99 + $1.99**
6	29 x 31*	15	$4.93 + 39c**
7	33 x 88*	16	7.41 – 2·5
8	1000 x 945	17	0.25 x 800
9	10% of 750	18	3.5 ÷ 0·5

* Items used by Reys, Reys & Hope (1993) and by Price (1997).
** Items were presented in a shopping context.

Results

The analysis of the data collected in this study is not yet complete. However, some preliminary results are presented here. Firstly, the computational preferences of students in four items used in two previous studies are examined. Then several selected interview extracts are presented to illustrate instances of how computational preferences are exercised by students, with particular attention to the use of a mix of methods.

Table 2
Percentage Distributions of Computational Choices Over Mental, Written and Calculator Use for Four Multiplication Items in Three Studies

Item	Swan & Bana n = 25			Reys, Reys & Hope n = 250			Price n = 18		
	M	W	C	M	W	C	M	W	C
36 x 25	20	40	40	20	71	9	3	70	27
70 x 600	48	8	44	39	45	16	41	33	26
29 x 31	12	64	24	29	63	7	5	56	39
33 x 88	12	56	32	13	73	14	3	66	31
All 4 items	23	42	35	25	63	12	13	56	31

Table 2 compares students' computational choices in the Reys, Reys and Hope (1993) study, the Price (1995) study, and the present study for four multiplication items. It should be noted that only broad comparisons can be made, as the items were administered differently in the three studies and the ages of the students also differed slightly. Also, only the US study by Reys et al used a large sample. It is interesting to note that the US study showed limited calculator use compared to the other two studies undertaken in Australian schools. The US students favoured written methods in all four items. However, this difference may simply be due to recency and sampling factors.

Further analysis of the results is still to be carried out but it should be noted there were several instances where students chose a particular method but were unsuccessful, then when asked if they could use another method they did so and found the correct solution.

Sample of Interview Extracts

The extracts which follow have been selected to illustrate the computational choices that students make and to show that these seem to fit a model of computation which includes a mixture of types, rather than a situation where students follow one particular method.

The first extract shows that Noel in Year 7 used a blend of calculator and mental methods to solve the problem.

Interviewer: Two thirds of 45
Noel: 15
I: 15 is it?
N: No it would be 30.
I: How did you do it?
N: I did 45 divided by three on the calculator to find one third and then doubled it in my head.

Several students adopted a similar approach The same item evoked a different response from Tim in Year 7 who completed the calculation 45 ÷ 3 on paper and then completed the calculation mentally and wrote 15 = 30, indicating the mental leap to the solution once he had established what one-third of 45 was by using the written algorithm.

The following student, Kylie from Year 7, demonstrated an ability to complete the same item in number of ways. For example, the item '369 ÷ 3' produced two different responses from her.

I: Three hundred and sixty-nine divided by three.
K: One hundred and twenty-three.
I: I noticed that you used a calculator, why?
K: Because it was easier.
I: Could you do it another way?
K: Yes — written

She then went on to demonstrate a perfectly executed short division algorithm. Clearly this student had at least two methods at her disposal and made the choice to use a calculator. She also demonstrated this choice in other items. For example, when confronted with the in-context question, "I went to the shop and bought two bottles of Pepsi at $1.99 each, so how much did they cost?", Kylie gave the following explanation.

K: $3.98 (after obviously undertaking mental calculations, then using a calculator)
I: I noticed that you tried to do it in your head to start with, and then you used your calculator. Can you tell me why?
K: I was getting confused with the numbers. I had put 2 and 2 together and got 4, and took 2 away and got 3.98, so I just checked it on the calculator.

The researcher had made the assumption that the student could not perform the calculation mentally and so had reached for the calculator. Asking the student to explain what she had done revealed a different picture.

Some students appear to like the security or familiarity of the written algorithm. In response to the item 369 ÷ 3 Linda in Year 7 wrote the algorithm down and the answer almost simultaneously.

I: That was pretty quick. I noticed that you wrote it down, but what was going on in your head?
L: As soon as I saw 369, well 1 x 3 is 3, 2 x 3 is 6 and 3 x 3 is 9.

It appears that some students do monitor what is happening in a calculation and adopt different approaches accordingly. The item '33 x 88' drew the following interesting response. A student wrote down 33 x 88 in the traditional format for the multiplication algorithm and ended with an answer of 44. He then changed his mind, reached for a calculator and computed the correct answer. When asked to explain why he had done this he had difficulty explaining his action and eventually replied, "because it was just quicker".

At times students seemed reluctant to express their thoughts for fear of giving an incorrect answer. In response to the item '10% of 750' the student referred to above used a calculator but did not make use of the percentage key. He entered 750 ÷ 10 into the calculator. When asked about this he said, "I did it in my head, but then I just checked it on the calculator".

The interviews revealed some gaps in student understanding. Sharon in Year 6 experienced a great deal of difficulty with subtraction. In her case she probably did not have any computational choice other than to use a calculator. Her comments below indicate that she does not have any mental or written strategies, other than an inefficient counting method, and is therefore restricted in her computational choice.

I: I noticed that you started writing strokes on paper and then you changed your mind and went for the calculator, why?
S: It just takes too long. I was going to write down 74 and then strike out 36 and then count.

The following extract from an interview with Alex in Year 6 highlights a mix between mental computation and his informal jottings on paper when the burdens on his short-term memory become too great.

I: Thirty-six times twenty-five
A: Nine hundred
I: Now you seemed to do most of it mentally there, but I notice that you wrote two things down. Tell me what happened.
A: I was a bit confused, there was too much to remember. First I started off with 2, 2 times 6 equals 12, and 1 is left over, so 2 times 3 equals 6, add 1 equals 7, so that's 72, so I added a 0, that's 720 (which he wrote down), and then I times, 5 times 6 equals 30, then 0 take 3, put the three where the other 3 is, then 5 times 3 equals 15, and the 3 is 18, so that's 180 (which he wrote down) and add it up.

Alex used a similar method when tackling the item '29 x 31'.

I: What were you doing it in your head when you wrote down 870 and 29?
A: 3 times 9 is 27, then you leave the 7, take the other 2, and 3 times 2 equals 6, add 2 equals 8, that's 87, and I added a zero because that's in the tens column, so I wrote it down there, so 1 times 29 is 29.

Likewise, Steven in Year 6 mixed mental methods and informal jottings to complete the item '28 + 37'.

I: Okay, you wrote down 50, then you wrote down 65, what were you doing?
S: I just added 20 and 30 together, and then I added 7 and 8 together.

A mix of calculator and written algorithm approaches was noted in the item '33 x 88'. A few students used the calculator to multiply three by eight, which is of concern, and then they went on to complete the written algorithm from that point. It should be noted that students sometimes chose unsuccessful methods. For example one student used mental computation to determine the answer to 369 ÷ 3 and gave an answer of 70. When asked whether she could do the calculation another way she used a calculator and achieved the correct result.

It appears that in some cases when the students lose confidence they reach for a calculator. For example one boy in response to the item '70 x 600' wrote down 600 x 70, decided he couldn't do it and then reached for a calculator. He could not explain why he couldn't do it on paper but was convinced that he couldn't. Perhaps he wasn't sure what to do with all the zeroes. Another student tackling the same problem tried using a mental method but ended up using a written algorithm. When asked about this his response was "I was just thinking, times that by 100, and then taking a couple off that, but I kept getting mixed up".

CONCLUSIONS AND RECOMMENDATIONS

As stated above, the data analysis for this study is still on-going. However, the initial findings based on the preliminary data from the 25 students drawn from Years 5, 6 and 7 attending a single primary school suggest the following tentative conclusions.

- Students often do not have a range of computational methods at their disposal. Some students do not have the luxury of a choice because they only know one method and therefore that method is their only choice. For example many students did not know how to perform a percentage calculation using a calculator and therefore this method of solution was not available to them.
- Students often start performing a calculation in one way using a particular computational method and then change part way through the calculation.
- When students lose confidence or realise they have made a mistake they will often use a calculator to complete the calculation or to check their working.
- Students seem to use the method that they feel most safe with. They might know how to do it another way but prefer to take the safe route.
- Some students make inappropriate choices because the method they use is unsuccessful in that they cannot complete the computation or it leads to an incorrect result.

These initial findings give a glimpse of the choices students do make and indicate that these choices are not simply split between three alternatives, but in some cases involve a combination of methods. The chosen methods are often idiosyncratic, depending on the item and on the individual. This suggests that the vast amount of time spent on teaching particular computational procedures in classrooms — whether mental, written or calculator methods — is likely to be of very limited use in the promotion of computational facility or number sense in students. It is suggested that exposing students to a variety of approaches and encouraging them to take responsibility for their own learning will be more beneficial. This could enhance students' choices of methods as well as increase their confidence in the computation process.

More research is needed to determine what factors affect students' computational preferences. There is evidence that the teaching program has some effect, particularly in reference to using a calculator. The magnitude of such an effect should be investigated through longitudinal studies.

REFERENCES

Australian Education Council. (1991). *A national statement on mathematics for Australian schools*. Carlton, Victoria: Curriculum Corporation.

Coburn, T. G. (1989). The role of computation in the changing mathematics curriculum. In P. R. Trafton & A. P. Shulte (Eds), *New directions for elementary school mathematics* (pp. 43-56). Reston, VA: NCTM.

Carraher, T.N., Carraher, D.W. & Schliemann, A.D. (1985). Mathematics in the streets and in the schools. *British Journal of Developmental Psychology, 3*, 21-29.

National Council of Teachers of Mathematics.(1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

Price, P.S. (1995). Choices of computational method made by children in years 5 to 7 to solve multiplication questions. Unpublished masters thesis. Queensland University of Technology, Brisbane.

Price, P. (1997). Teacher presence as a variable in research into students' mathematical decision-making. In F. Biddulph & Ken Carr (Eds), *People in mathematics education* (Proceedings of the 20th annual conference of the Mathematics Education Research Group of Australasia, pp. 422-428). Rotorua, NZ: MERGA.

Porter, A. (1989). Curriculum out of balance: The case of elementary school mathematics. *Educational Researcher, 18*(5), 9-15.

Reys, R. E., Reys, B., & Hope, J. (1993). Mental computation: A snapshot of second, fifth and seventh grade student performance. *School Science and Mathematics. 93*(6), 306-315.

Ruthven, K. (1995). Pupils' views of number work and calculators. *Educational Research, 37*(3), 229-237.

Ruthven, K. (1998). The use of mental, written and calculator strategies of numerical computation by upper primary pupils within a 'calculator-aware' number curriculum. *British Educational Research Journal, 24*(1), 21-42.

Swan, P. and Bana, J. (1998) Student choice of computation methods. In C. Kanes, M. Goos & E. Warren (Eds), *Teaching mathematics in new times* (Proceedings of the 21st annual conference of the Mathematics Education Research Group of Australasia, pp. 580-587). Brisbane: MERGA.

Trafton, P. (1994). Computational estimation: Curriculum development efforts and instructional issues. In Reys, R. E. & Nohda, N. (Eds), *Computational alternatives for the twenty-first century: Cross-cultural perspectives from Japan and the United States* (pp. 76-86). Reston, VA: NCTM.